TEACHER FEEDBACK AND AUTHORITY DURING INSTANCES OF STUDENTS' PROVING

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Teacher feedback during the process of students' proving is important to consider because proving is often challenging for students, requiring feedback and support. Additionally, feedback has implications for authority and agency, which are constantly being negotiated. We examined the authority dynamics evidenced in a teacher's feedback actions while students proved geometry claims. By analyzing audio and video recordings of classroom proving discussions, we found that various teacher feedback types position either the teacher or students as authorities with regard to providing and validating mathematical ideas. We provide suggestions for research and practice with respect to teacher feedback and authority in proof instruction.

Keywords: Proving, Authority, Classroom Discourse, Instructional Activities and Practices

Introduction

Mathematics proving has long been considered a central component of secondary students' mathematics (Schoenfeld, 1994; Stylianides et. al., 2017) because, among other reasons, it is useful as a process through which students make sense of and communicate mathematical knowledge (de Villiers, 1999; NCTM, 2000). Despite the significance of proving in mathematics learning, research evidence shows that secondary school students struggle with constructing and understanding proofs (Chazan, 1993; Harel & Sowder, 2007; Shongwe, 2020). This tension highlights the need to examine how teachers can support their students in learning this important yet difficult concept. Scholars have studied some teacher practices like the careful enactment of proof tasks (Bieda, 2010) and teacher moves (Martin et.al., 2005) to support students' learning of proving. The present study builds on the work of these researchers by examining how teacher feedback practices can support students' as they learn to construct a proof.

Teacher feedback is defined as the information provided by a teacher to students about their performance or understanding (Dempsey, et al., 1993; Hattie & Timperley, 2007) and is known to support student learning in a variety of ways. Feedback provides information that learners can use to confirm, reject, or modify prior knowledge (Fyfe et al., 2015), increases student motivation, and acts as a guide for what students should do to make progress towards the learning goals (Hattie & Timperley, 2007). Hattie and Timperely, however, caution that the effectiveness of feedback depends on the type of feedback and the way it is given, adding that feedback by itself may not have the power to initiate student action because students can accept, modify, or reject teachers' feedback.

Beyond academic performance, another way to examine the power of feedback is by focusing on how it promotes or hinders student authority in classroom learning. Powerful classrooms are ones where students provide the ideas that drive classroom discourse (Engle & Conant, 2002). Such classrooms are characterized by students having opportunities to talk elaboratively (Soter et al., 2008) and actively participate in the class by taking up positions of authority (Engle, 2012; Otten et al., 2017). Students' authority and active participation in classroom proving are important because they allow for student ownership of the proving

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process with students learning to make and justify conjectures (Otten et al., 2020) without relying on the teacher to always tell them what to do. In other words, feedback can be a way for a teacher to support a student and spur them forward or focus their attention without explicitly directing their actions. For these reasons, the present study considers authority in the context of students' proving, with the object of study being the teacher's feedback actions.

Theoretical Framing

Our view of proofs is guided by the work of de Villiers (1999), who viewed proofs as a tool to accomplish several purposes such as verification, explanation, communication, and intellectual challenge. These purposes are not solely for individuals but function within a classroom community in which teachers have a dual duty to represent the discipline of mathematics as a more knowledgeable other while also honoring students as learners of mathematics and attending to their needs (Stylianides, 2007). We connect these ideas to teacher feedback which can be a tangible act through which a teacher responds to students' learning needs while also representing mathematics in an authentic way.

Proving in a classroom community inculcates a discourse, during which patterns of interaction and authority dynamics are continuously established and negotiated (Otten et al., 2017). For example, teachers and students may negotiate what definition(s) and format(s) to use during classroom proving and there may be a negotiation over whether a proof is sufficiently complete or clear. During classroom negotiations, the person or object in authority is the one who takes the lead while others follow (Pace & Hemmings, 2007). Using this definition, we view the person in authority as the one who leads the proving discourse by deciding what ideas are foregrounded and validated.

Teacher's feedback may either maintain the authority of teachers or position students in authority of leading the proving discourse. We investigated the authority dynamics manifested during teacher feedback by examining both the focus and purposes of the feedback. In terms of focus, Hattie and Timperley (2007) outline that teacher feedback may focus on task correctness, students' processing of information and/or students' motivation. Dempsey and colleagues (1993) on the other hand delineate various feedback types that play different purposes (e.g., providing correct answers, probing students to think towards the correct answers, among others) during classroom learning. To this end, we ask two key questions:

RQ1) What feedback types does a teacher provide students during classroom proving?

RQ2) How is authority manifested in the feedback types used by this teacher?

Method

Setting and Participants

Data for this study came from a larger teaching experiment that explored a non-traditional way of introducing proof to secondary students. The teaching experiment involved students using tasks and strategies that attended to the generality and purpose of proofs rather than direct instruction on the specific techniques for constructing arguments (Conner, 2018). The third author was the instructor in this teaching experiment that involved ten students (7 females; 3 males) enrolled in an accelerated 9th grade mathematics course at a rural, public school in the Midwest United States. Prior to the study, these students had not received any high school Geometry or formal proof instruction, although reasoning and justification had been a part of earlier mathematics courses. The teaching experiment consisted of 14 sessions (ap539anderb28–38 minutes each), with students primarily working on tasks in three small groups. This study

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focuses on Sessions 11-13, when students were engaged in conjecturing and proving claims about specific classes of polygons and whether or not they are all similar (e.g., "all squares are similar"). The instructor used non-traditional instructional practices because she hoped to share authority of proving with students by actively engaging them in both the discourse and practice of reasoning-and-proving and she intended for students to see the purpose of deductive reasoning instead of relying on teacher authority to direct them to use deductive rather than empirical reasoning when constructing proofs.

Data Sources and Analysis

As part of the larger study, all classroom discourse was audio and video recorded with recorders at each of the three groups as well as focused on the full class setting. Students' written work was also collected. Selected sections of the classroom discourse (i.e., when there were interactions between the teacher and students) were transcribed and coded using MAXQDA software. We coded for oral teacher feedback given during both whole-class and small-group discussions. For this report we are not including instances of student-to-student feedback, although this did occur.

Data analysis comprised three phases of coding. Phase one involved flagging any instance of teacher-to-student discourse that contained (or functioned as) feedback to the students about their performance, understanding, or directing them toward or away from a solution path. We tried to be overly inclusive in this flagging of teacher feedback. In phase two we coded for feedback types based on the works of Dempsey and colleagues (1993) and Hattie and Timperley (2007):

- *Knowledge of results feedback* - –informs learner whether their strategy or answer is correct or incorrect (e.g., "that is a good guess, but it is not correct").
- *Knowledge of correct response feedback* - –informs the learner what the correct strategy or answer is (e.g., "the angles of an equilateral triangle are all the same size").
- *Elaborated feedback on correct response* - –gives explanations for why the student's response is correct and/or directs students to relevant materials or information that could strengthen the response (e.g., We can label the angles of a square with a representation of 90 degrees because the angles are always going to be 90 degrees).
- *Elaborated feedback on incorrect response* - gives explanations for why the student's' response is incorrect and/or directs them to relevant materials or information that could lead to a correct response (e.g., "what you provided is a conditional statement, but it's not a conditional statement for squares being similar").
- *Questioning feedback* - –poses questions to respond to students' strategy or answer with the questions functioning as an indicator of correctness or incorrectness (e.g., "[Student X] said that isosceles triangles are similar. What do you all think?").
- *Revoicing feedback* – Uses a restatement of students' strategy or answer to respond to students with the restatement functioning as an indicator of correctness or incorrectness *(*e.g., "[Student Y] just gave us a counterexample. He said …").

The *questioning* and *revoicing* feedback codes were not in the prior literature but emerged from our analysis because these teacher moves, in some instances, functioned as a form of feedback to students. Note, however, that not all teacher questions nor all instances of teacher revoicing were necessarily feedback. For example, to start Session 11, the teacher asked students to read the task description then revoiced what the students read. In this context, this question and revoicing did not indicate to the student any information about their performance and was thus not coded.

Finally, in phase three, we coded for mathematical authority dynamics using our definition of authority. We examined the authority at play for each instance of coded feedback by asking the following questions: Who provided the mathematical ideas that were the focus of the discourse? Who critiqued or validated the correctness of the mathematical ideas? And who confirmed or rejected the completeness and correctness of proofs? We considered these questions with respect to the teacher and to students collectively (not individual students). We then considered patterns across instances in terms of the authority for each feedback type. Multiple authors analyzed the data and met regularly to clarify and reconcile coding differences.

Findings

The classroom proving discourse was marked by various feedback types, with questioning feedback being the most common. Table 1 shows the number of instances of each feedback type and the predominant authority figure for each feedback type.

Table 1: Authority Figure(s) and Number of Instances for Various Feedback Types

Feedback Type	Number of Instances	Predominant Authority Figure
Knowledge of results	9	Teacher
Knowledge of correct response	12	Teacher
Elaborated feedback for correct response	9	Student
Elaborated feedback for incorrect response	5	Teacher
Questioning feedback	24	Student
Revoicing feedback	13	Student

Authority dynamics in the feedback were two-fold. There was authority in terms of who provided mathematics ideas that propelled the classroom proving, and authority in terms of who validated the correctness of the proofs. Our findings show that *knowledge of results*, *knowledge of correct response*, and *elaborated feedback for incorrect responses* positioned the teacher as authority whereas *questioning feedback*, *revoicing feedback*, and *elaborated feedback for correct responses* positioned students as authority.

Feedback Types that Position the Teacher as Authority

Knowledge of results feedback positioned the teacher as the mathematical authority with regard to deciding whether students' responses or their steps within the argument they were formulating were mathematically correct. Moreover, through *knowledge of correct response* feedback, the teacher positioned herself in authority of both providing what counted as true responses and the ideas that moved the proving process forward. For example, consider the excerpt below from a whole-class discussion at the start of Session 12 when the teacher was introducing conditional statements.

T: To start off, does anyone remember anything about conditional statements? We talked about them when we were doing the diagrams? No? Okay.

S: They were true sometimes but not all the time, right? They were not for sure things.

T: That would be a good guess, but no. That is not what it is (*chuckles*). A conditional statement is just a statement that is written in a certain form. It is written in the form 'if something, then something. (*Writes the statement if _ then _ on the white board.*)

Here, a student attempted to give the definition of a conditional statement, and the teacher gave

knowledge of results feedback, “that is not what it is.” In this case, the teacher positioned herself in authority of determining that the students’ response was incorrect. The teacher then went on to provide *knowledge of correct response* feedback on what the correct response was by saying, “A conditional statement is just a statement that is written in a certain form.” Again, the teacher claimed authority in giving the correct response and also providing the definition which would be used later in proving activities.

Later in this class as the students were attempting to use the earlier given definition of conditional statements to prove that ‘all squares are similar,’ the teacher gave *knowledge of results, knowledge of correct response*, and *elaborated feedback for incorrect response*, again positioning herself as the mathematical authority. See the discussion below:

T: Can anyone take a stab at writing our statement ‘all squares are similar’ into a conditional if-then kind of format? (*Looking at student S1 specifically*) You want to take a stab?

S1: ..If all the angles on a square are 90 degrees, then they are all the same.

T: So that is a conditional statement, an if-then, but it is not a conditional statement for squares being similar. So, what could we--, what did we assume to be true when we were proving our statement about the squares being similar? What did we start off with?

S2: That they were similar?

T: That is what we are trying to prove. So, we did not start off with that.

S3: That they have four 90 degrees angles?

T: So that is actually a property of squares. So, we just started with, “if we have squares.”

When Student 1 gave a guess of how to write a conditional statement for all squares being similar, the teacher used both *knowledge of correct response feedback* and *elaborated feedback for incorrect response* to tell the students why their response was incorrect (“that is a conditional statement ..It it is not..Ir squares...”). Here the teacher positioned herself in authority, determining what a correct conditional statement for all squares being similar would be. The teacher then rephrased the question by asking, “What did we start off with?” Again, when two students gave the responses “they were similar” and “they have four-90 degrees angles,” the teacher gave *elaborated feedback*, wherein she did not explicitly tell the student that their responses were incorrect but rather elaborated the incorrectness by explaining that the responses were what needed to be proved or just “a property of squares.” Finally, the teacher gave *knowledge of correct response feedback* by stating that “we just started with, ‘if we have squares.’” In this way, the teacher assumed authority in validating the correctness of what the class “start[ed] off with” and in providing the correct response.

Feedback Types that Position the Students as Authority

Feedback that was in the form of *questioning*, *revoicing*, and *elaboration of correct responses* tended to position the students in authority of providing correct responses and ideas that moved the classroom proving discussion forward. For instance, see the discourse below when students were attempting to come up with conjectures for similar polygons at the beginning of Session 11:

S1: Is a circle a polygon?

T: Huuh. That was the question over there too. (*Asking the whole class*) Okay. Quick question. Is a circle a polygon?
S2: Yes.
S3: No.
S4: It has no defined sides.

In this case, the teacher avoided taking authority by giving the students the correct answer, and instead used *questioning* and *revoicing feedback* simultaneously by posing the same question to the whole class, thus positioning all students in authority of offering thoughts and potentially deciding as a group what they thought the correct answer to be. Indeed, a whole-class discussion continued until all students seemed to agree that circles are not polygons.

A similar authority dynamic happened in another instance during a small-group discussion in Session 11 when students were attempting to formulate a conjecture about specific triangles that are similar. One student asked the teacher, "All equilateral triangles are similar because they have 90 degrees--, no, which one has a 90-degrees angle?" The teacher responded using both *questioning* and *revoicing feedback* by re-asking the question to the other students in that group, "which one has 90 degrees?" again positioning students in authority of deciding together what could count as correct responses.

The prior examples involved shared authority as students clarified the scope of what they should consider in their proving process. The teacher also used *elaboration feedback on correct responses* in addition to *questioning* and *revoicing feedback* to position the students in authority of providing ideas that would propel the proving discourse forward. For example, see the excerpt below from Session 13 when the whole class was discussing how to label two squares before proving that they are similar.

T: Now, you all had a couple of different ways of correctly labelling this diagram, so we are going to talk about it. But first of all, let's talk about the angles (*the teacher has two unlabeled squares on the board*). What do we know, or how do we label the angles of our square?
S1: Put a box.
T: Put a box. And what does the box represent or tell us?
S2: 90 degrees.
T: 90 degrees. Now why can we label it 90 degrees instead of using like a letter?
S3: It's always going to be 90 degrees.
T: It is always going to be 90 degrees, because we are talking about squares.

In this example, the teacher started by giving a general *elaboration feedback* on students' previous correct work of labeling diagrams and invited them to share what they did. During the discussion, the teacher used both *revoicing feedback* to foreground what the students said and did so in a way that seemed to indicate the students were correct, and the teacher used *questioning feedback* (e.g., what "the box" means in angles and why we label squares using a "box") to solicit ideas from the students. This way, although the teacher was guiding the discussion, she centered students' responses, thus sharing with them the mathematical authority of giving ideas that propelled the classroom proving discourse.

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Discussion and Conclusion

This study aimed to determine the teacher feedback types given to students during classroom proving and to examine the authority dynamics of the various feedback types. The teacher gave a variety of feedback types that positioned either the teacher or the students as mathematical authorities. The feedback types that tended to position the teacher in authority were those where the teacher gave verdict on the correctness of students' responses and where the teacher provided the students with the correct responses that moved the proving discourse forward. The feedback types that positioned the students in authority were mostly through questions that probed students to think deeper or questions that invited students to respond to each other's ideas and hence develop a shared notion of how the classroom proving process might continue.

It is our view that the variety of teacher feedback maintained a balance of authority between the teacher and students. This form of balance is worthwhile to consider because although it may be viewed as ideal for students to have more agency, giving entire authority to students is not always feasible, especially when they are learning formal proving for the first time (Otten et al., 2020). There are times when it is rational for the teacher to take up the role of the more knowledgeable other by providing corrective guidance to students. For example, there are incorrect or unproductive ways to turn the conjecture "all squares are similar" into a conditional statement and these could have negative implications for the proof the students were about to construct.

Our study contributes to the ongoing efforts to support teachers in sharing authority with students as a way of enhancing students' meaningful learning of math and active participation (Engle, 2012; Otten et al., 2020). Sharing authority in areas like proving may seem challenging due to the inherently complex nature of teaching and the difficulties students experience with learning proofs (Chazan, 1993; Harel & Sowder, 2007; Shongwe, 2020). We nevertheless encourage teachers to enact feedback practices that invite students to share in the authority of mathematical ideas to encourage students to become more adept at conjecturing, making arguments, and critiquing the arguments of others. Elsewhere, we encourage teachers to share authority gradually and strategically with students (see Otten et al., 2020) in the spirit of maintaining the dual role of teaching in a classroom community (Stylianides, 2007). Our findings in this study show that questioning feedback could be another strategy for sharing proving authority with students.

Finally, using feedback as a lens for viewing questioning is another way to think about the literature on questioning. Questioning has been documented as a teacher practice that is essential in promoting active student participation (Black et al., 2003). When used as teacher feedback, questions can invite students to take up authority of their own learning and that of their peers. Through questions, teachers can assess student thinking but also provide implicit feedback guiding them towards the learning goal. Good questions spur students to confirm or modify their prior thinking, detect errors and correct them without the teacher explicitly telling them the correct answer. Questions also serve to invite students to respond to and critique each other's arguments, thus promoting rich classroom proving discussions. For example, when the teacher re-asked a student's question on whether circles are polygons back to the class, the students held a discussion until they agreed that circles are not polygons. Future research might document the differential outcomes between a teacher providing directive feedback and questioning feedback, with questioning feedback not only having the possibility of promoting shared authority but also being aligned with the kinds of discourse that we hope to be common in the proving process.

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